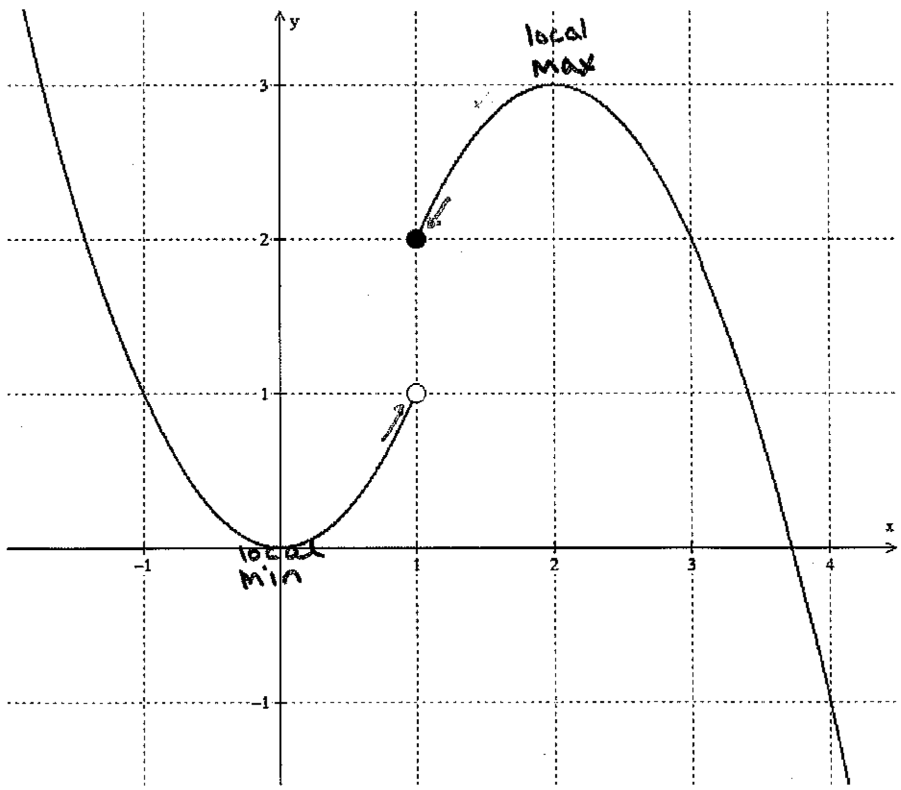
Pre Calculus Honors **Name:**
2014 Final Exam Review #1 **Date:**

Functions & Polynomials



1. Write the **domain** and **range** for the graph of f(x) in interval notation Domain : $(-\infty, +\infty)$ $(-\infty, 1)$ $[1, +\infty)$ Range: $(-\infty, +\infty)$	2. Label the relative maxima and minima
3. Find the values for: f(-1) = 1 f(2) = 3	4. For what x values does f(x) = 0? (Estimate one of them.) (2 pt) x = 0 & x = 3.75 For what x values does f(x) = 2? (2 pt) x = 1 x = 3 x = -1.7
5. Identify the intervals where the graph is increasing. (0, 2)	6. Use the graph to find the following limits $\lim_{x\to1+} f(x)$ = 2 $\lim_{x\to1-} f(x)$ = 1 $\lim_{x\to1} f(x)$ = DNE

7. Given $f(x) = 5x^2 - 3x + 7$ and $g(x) = 5 - x^3$, write **simplified** expressions for (2 pts each):

a) $f(g(x))$

$5(5-x^3)^2-3(5-x^3)+7$

$5x^6-47x^3+117$

b) $f(x)-g(x)$

$5x^2-3x+7-(5-x^3)$

x^3+5x^2-3x+2

c) $f(x)g(x)$

$(5x^2-3x+7)(5-x^3)$

$25x^2-5x^5-15x+3x^4+35-7x^3$

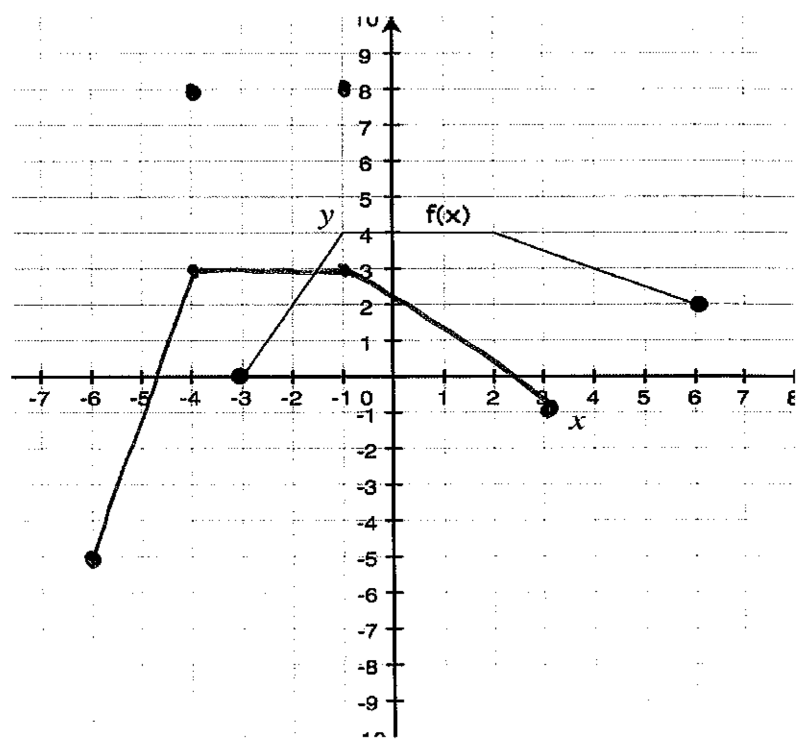
8. The graph of the function $f(x) = 7x^2 + 3$ is shifted 8 spaces to the left and 7 spaces down. What is the equation of the new graph? (2 pt)

$f(x) = 7(x+8)^2-4$

stretch horizontal shift

vertical shift

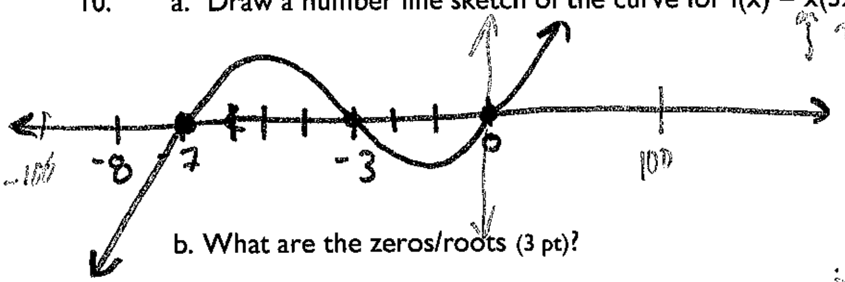
9. On the same axes, draw a graph of $g(x) = 2f(x + 3) - 5$ (3 pt)

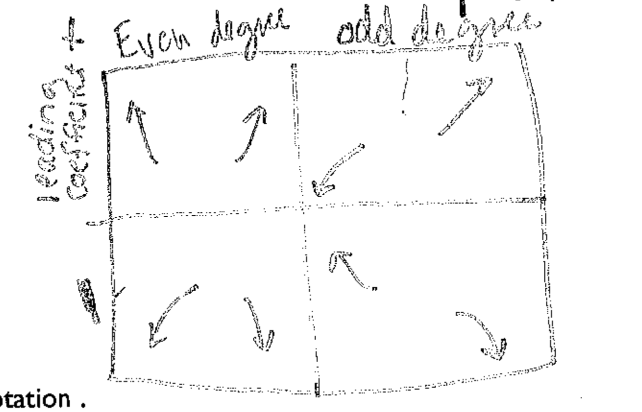


10. a. Draw a number line sketch of the curve for $f(x) = x^{1}(3x+9)^{1}(x+7)^{1}$.

Degree: 3

Leading coefficient: positive





b. What are the zeros/roots (3 pt)?

$x = 0$

$x = -3$

$x = -7$

c. Describe the end behavior using proper interval notation .

As $x \to +\infty$, $y \to +\infty$

As $x \to -\infty$, $y \to -\infty$

d.. State the interval (in proper interval notation!) where $f(x) \leq 0$ (3 pt).

$y \leq 0$

$[-3, 0]$

$(-\infty, -7]$

11. Solve: $2x^2 - 7x + 8 = x^2 - 8x + 50$ (3 pt)

$x^2 + x - 42 = 0$

$(x+7)(x-6) = 0$

$x = -7 \quad x = 6$

12. Reduce (3 pt) : $\frac{x^2-11x+18}{x^2+2x-8}$

$$\frac{(x-9)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$$

$$\frac{x-9}{x+4}$$

13. Combine the polynomial fractions (3 pt): $\frac{5x+7}{x+1} - \frac{x+3}{x+2}$

$$\frac{(x+2)}{(x+2)}\frac{5x+7}{x+1} - \frac{x+3}{x+2}\frac{(x+1)}{(x+1)}$$

$$\frac{4x^2+13x+11}{(x+1)(x+2)}$$

14. For the function $\frac{x^2-5x-14}{x^2+4x+4} = \frac{(x-7)(x+2)}{(x+2)(x+2)}$

a. Find the x-intercept (2 pt) $x^2-5x-14=0$ $(x-7)(x+2)=0$ $x=7$

b. Find the y-intercept (2 pt) $y=-\frac{7}{2}$

c. What x-value has a hole? (2 pt) $x=-2$

d. What x-value has a vertical asymptote? (2 pt)

$x=-2$

- x-int. when the numerator is zero.
- y-int. occur when you plug in 0 for x.
- holes occur when there's a common factor so that you get $\frac{0}{0}$
- V.A. When ~~does~~ just the denominator equals 0.

Logarithms

15. Rewrite the logarithms as exponential equations (1 pt each).

a. $\log_3(81) = 4$

$3^4 = 81$

b. $\log_{289} 17 = \frac{1}{2}$

$289^{1/2} = 17$

c. $\log_6(36)=2$

$6^2 = 36$

16. Simplify into a single logarithm using logarithm rules (2 pt):	17. Expand using logarithm rules (2 pt):
$\mathbf{log_4}\, x - 6\mathbf{log_4}\, y$ $\log_4 \frac{x}{y^6}$	$\log(\frac{10x^2}{8})$ $1 + 2\log x - \log 8$

18. Solve using logarithm rules (1 pt each):

a. $\ln(e^4) =$ 4

b. $\log_b(b) =$ 1

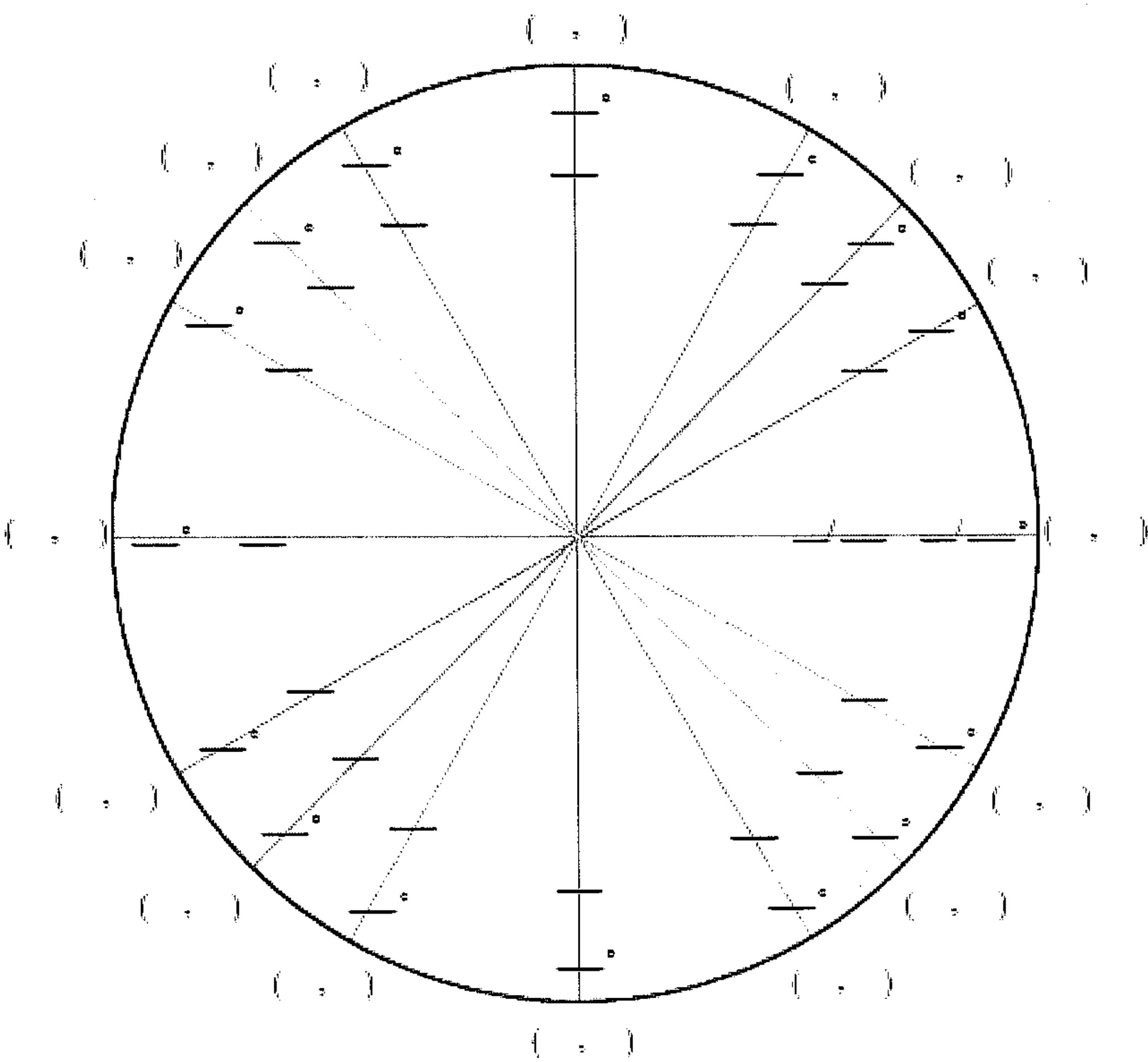
19-22 Solve the exponential equations using logarithms and vice-versa. Show all work.

19. $16^{n-7} + 5 = 24$	20. $11^{n-8} - 5 = 54$
$n = 8.062$	9.70047

21. $-2 \log_8(x+1) = -8$	22. $\log_6(-3m-1) = \log_6(-4m-6)$
$8^4 = x+1$ $x = 4095$	$-3m-1 = -4m-6$ $m = -5$

Unit Circle

23. Complete the Unit Circle by filling in the degrees, radians, and coordinate points. You may tear this page to use as a reference. Staple it to the end of the test.



24 – 35 Find the solution using your U.C. READ ALL QUESTIONS CAREFULLY!

24. $\sin\left(\frac{\pi}{3}\right)$ $\frac{\sqrt{3}}{2}$

25. $\cos\left(\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{2}$

26. $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

27. $\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}}$

28. $\sec\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$

29. $\tan(\frac{4\pi}{3})$

30. $\sin(\frac{11\pi}{6})$

31. $\cos(\pi) = -1$

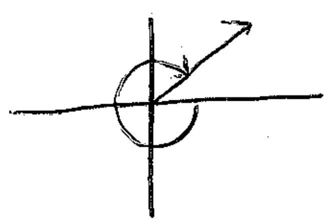
32. $\sin(0) = 0$

33. $\cos^{-1}(-\frac{1}{2})$

34. $\arcsin(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$

35. $\tan^{-1}(1) = \frac{\pi}{4}$

36. Sketch a $-7\pi/4$ radian angle in standard position and *identify the quadrant* the terminal side falls in(1 pt).



Quad 1

37. Convert 312° to radians. Your answer should be an exact answer and should be reduced (2 pt).

$$312^\circ \cdot \frac{\pi}{180^\circ} = \frac{78\pi}{45}$$

38. Find a coterminal angle between 0 and 2π radians for the angle $4\pi/3$ (1 pt).

$\frac{4\pi}{3}$ is betwen 0 & 2π already!

39. In a ***right triangle***:

a. $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

b. $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

c. $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

40. On the ***unit circle***:

a. $\sin\theta = y$

b. $\cos\theta = x$

c. $\tan\theta = \dfrac{y}{x}$

41. The **reciprocal trig functions** are defined as (in relation to sin/cos/tan) (4 pts):

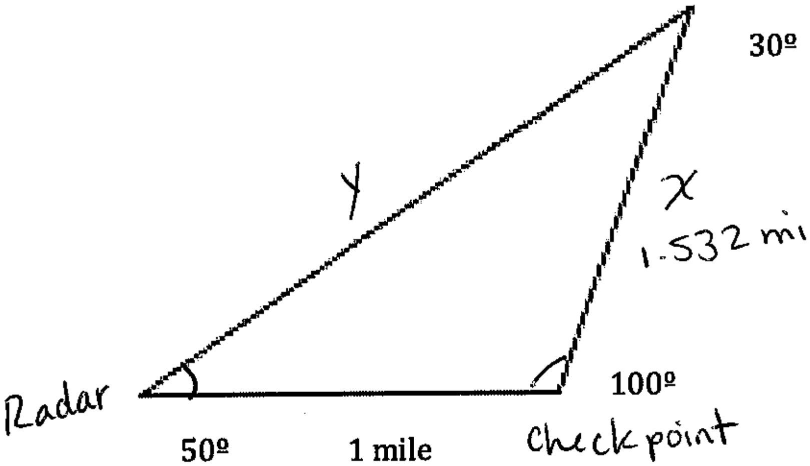
a. cosecant $\dfrac{1}{\sin\theta} = \csc\theta$

b. secant $\dfrac{1}{\cos\theta} = \sec\theta$

c. cotangent (write two definitions)

$$\frac{1}{\tan\theta} \text{ or } \frac{\cos\theta}{\sin\theta} = \cot\theta$$

42. An airplane flies over a radar station and then a checkpoint 1 mile away, both located on level ground. At the moment the angle of elevation of the airplane above the radar station is 50° and the angle between the station and checkpoint is 30°, find the distance between the airplane and the checkpoint using the Law of Sines, and then find the distance between the airplane and the radar station using the Law of Cosines. (6 pt)



$$\frac{\sin 30^\circ}{1} = \frac{\sin 50}{x}$$

$x = 1.532$ miles

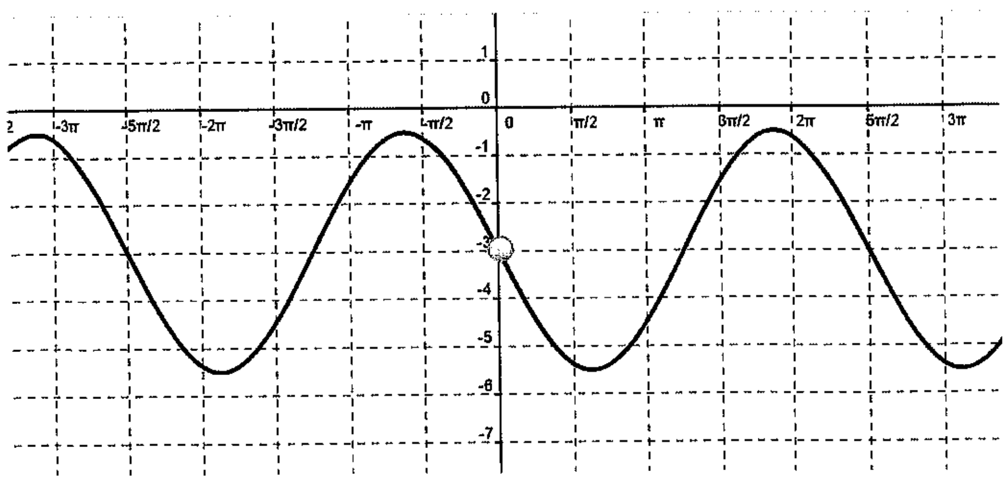
$y = 3.77$ miles

43. Find the area of the triangular lot having 2 sides of lengths 90 meters and 52 meters. These two sides form an angle of 102°. (3 pt) $2288.87\ m^2$

$A = \frac{1}{2}ab\sin C$

44.

Identify the parent function, amplitude, vertical shift, and period of the function graphed below. *It has not been phase shifted.* Write an equation.



Amplitude: 2.5

Vertical Shift: -3

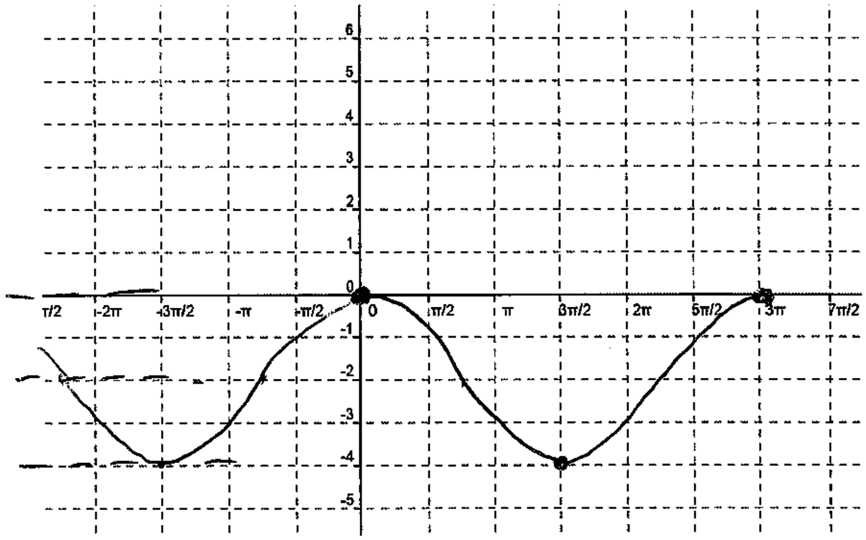
Period: $2\pi \div \frac{5\pi}{2} = \frac{4}{5}$

Parent Function: sin

Equation : $2.5\sin\left(\frac{4}{5}(x+0)\right)-3$

45. Graph the equation. Identify all pieces listed.

$f(x) = 2\cos(\frac{2}{3}x) - 2$



Amplitude: 2

Vertical Shift: -2

Period: $\frac{2\pi}{2/3} = 3\pi$

Parent Function: cos

$\sin^2\theta + \cos^2\theta = 1$	$\cot^2\theta + 1 = \csc^2\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\cos^2\theta = 1 - \sin^2\theta$	$\cot^2\theta = \csc^2\theta - 1$	$\tan^2\theta = \sec^2\theta - 1$
$\sin^2\theta = 1 - \cos^2\theta$	$1 = \csc^2\theta - \cot^2\theta$	$1 = \sec^2\theta - \tan^2\theta$

46. Verify the following identity. **Show all steps of the verification** (2 pt)

$$\frac{\sec^2\theta - \tan^2\theta}{\sin\theta} = \csc\theta$$

$$\frac{1}{\sin\theta}$$

$\csc\theta = \csc\theta$ ✓

47. Verify the following identity. **Show all steps of the verification** (2 pt)

$$(\csc^2\theta - 1)\tan\theta = \cot\theta$$

$\cot^2\theta \cdot \tan\theta$

$\frac{1}{\tan^2\theta} \cdot \tan\theta$

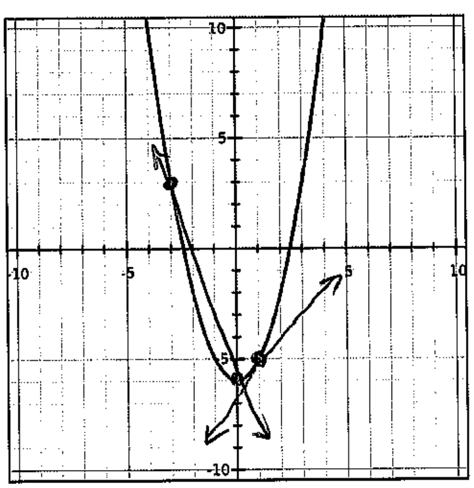
$\frac{1}{\tan\theta} = \cot\theta$ ✓

48. Verify the following identity. **Show all steps of the verification** (3 pt)

$$\frac{\sin\theta}{\cos\theta} + \frac{1}{\sin\theta} = \frac{\sin^2\theta + \cos\theta}{\sin\theta\cos\theta}$$

Tangents and Derivatives

49. Sketch and label a secant line. Sketch and label a tangent line on the following graph. (2 pt)



secant line – 2 pts

tangent line → 1 pt

50. Find the limit of $\lim\limits_{x \to 3}(2x+7)$ = 13

51. Find $\lim\limits_{x \to -2} \frac{x^2 - x - 6}{x^2 + x - 2}$ Direct substitution won't work, so simplify first!

$$\lim_{x \to -2} \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)} = \frac{5}{3}$$

52. Given the function f(x) = 3x² – 5x + 6 find the derivative/slope of the tangent line at x = 4, using the difference quotient $\lim\limits_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right) =$

$$f'(x) = 6x - 5$$

$$f'(4) = 19$$ ← slope of the tangent line!

53. Find the derivative (equation for the slope of the tangent line) of the function $y = \sqrt[3]{x} + \frac{2}{x}$

$$y = \frac{1}{3\sqrt[3]{x^2}} + \frac{-2}{x^2}$$